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EE 371
Engineering Electromagnetics I

Midterm Examination #1

Open Book

Answer All Questions

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1. Flux representations of vector fields provide a useful means for visualizing these fields and illustrating their variations in space. Draw the flux representations of the following vectors:

a. \( \mathbf{A} = \rho \mathbf{a}_z \)

b. \( \mathbf{C} = \sin \phi \mathbf{a}_\rho \)

c. Coulomb's electric field law: \( \mathbf{E} = \frac{Q}{4\pi \varepsilon_0 r^2} \mathbf{a}_r \)

d. Magnetic flux density \( \mathbf{B} \) from a long wire carrying a current \( I \): \( \mathbf{B} = \frac{\mu_0 I}{4\pi \rho} \mathbf{a}_\phi \)

\[ e. \quad \mathbf{F} = \sin \theta \sin \phi \mathbf{a}_r, \quad \text{for} \ \phi = \frac{\pi}{2} \]
2.a. A line integral of a force vector $\mathbf{F}$ of the form $\int \mathbf{F} \cdot d\mathbf{l}$ has the physical meaning of calculating the work done by the force while moving along the contour $c$. Explain this statement.

b. An electric field is given in the spherical coordinates by

$$\mathbf{E} = K \frac{\cos \theta}{r^3} \mathbf{a}_r + \frac{K \cos \theta}{2} \frac{1}{r^3} \mathbf{a}_\phi$$

Determine the work done on a unit positive charge in moving from point $a$ to point $b$ in the presence of the $\mathbf{E}$ field given above.

![Diagram of electric field and path](image)

**Fig. 1.** The path of the electric charge from $a$ to $b$.

$$\int_{a}^{b} \mathbf{F} \cdot d\mathbf{l} = \int_{a}^{a_1} \mathbf{E} \cdot d\mathbf{l} + \int_{a_1}^{a_2} \mathbf{E} \cdot d\mathbf{l} + \int_{a_2}^{b} \mathbf{E} \cdot d\mathbf{l}$$

1. $d\mathbf{l} \ (\text{for } a \to a_1) = r \, d\phi \, \mathbf{a}_\theta$
2. $d\mathbf{l} \ (\text{for } a_1 \to a_2) = r \, d\phi \, \mathbf{a}_\phi$
3. $d\mathbf{l} \ (\text{for } a_2 \to b) = dr \, \mathbf{a}_r$

Limit $\phi = \frac{2\pi}{5}$ to $0$

Then (9) on completing the calculations.
3. a. Gauss's law for the magnetic field $\int \mathbf{B} \cdot d\mathbf{s} = 0$ emphasizes two unique and important characteristics of magnetic fields. Select the two most important properties from the following list:

i. No single magnetic poles.

ii. Magnetic flux lines originate on north poles and terminate on south poles.

iii. Magnetic flux lines originate on south poles and terminate on north poles.

iv. Magnetic flux lines are closed lines.

v. Magnetic fields exist around magnets with north and south poles.

c. Show that the vector $\mathbf{B}$ given by $\mathbf{B} = \frac{1}{\rho} \mathbf{a}_\rho$ may represent a magnetic flux density vector. Simply show that it satisfies Gauss's law for the magnetic field by using the closed surface shown in Fig. 2.

![Diagram](image)

**Fig. 2.** Geometry of closed surface to be used in Gauss's law.
4. Consider the time-varying magnetic flux density vector \( \mathbf{B} \) given by

\[
B = 6 \sqrt{x^2 + y^2} \tan \phi \cos \omega t
\]

A rectangular conducting loop is placed at the \( \phi = \pi/3 \) plane as shown in Fig. 3. Determine:

![Diagram of a rectangular conducting loop placed at the \( \phi = \pi/3 \) plane.]

Fig. 3. A rectangular conducting loop placed at the \( \phi = \pi/3 \) plane.

a. The total magnetic flux \( \psi_m \) crossing the area of the loop.

b. The induced emf at the terminals of the loop.

c. Plot \( \psi_m \) and the induced emf as a function of time and show that the induced emf satisfy "Lenz's law."

\[
\psi_m = \int_{S_{0.09}} \mathbf{B} \cdot d\mathbf{s}
\]

\[
d\mathbf{s} = d\rho d\zeta \mathbf{a}_\phi
\]

\[
\psi_m = \int_{S_{0.09}} 6 \rho \tan \phi \cos \omega t \mathbf{a}_\phi \cdot d\rho d\zeta \mathbf{a}_\phi = \frac{6 \rho \tan \frac{\pi}{3}}{3} \cos \omega t \int d\rho d\zeta
\]

\[
\rho = 0.05 \quad \rho = 0.06
\]

\[
\text{emf} = -\frac{\partial \psi_m}{\partial t} = -\left( -\omega \sin \omega t \right) \left( -\omega \cos \omega t \right) = \omega \sin \omega t
\]
(+): Means resulting magnetic flux is in the direction of the original decreasing magnetic flux (trying to prevent the decrease).

(−): Means resulting magnetic flux from the induced emf is in the opposite direction of the original increasing magnetic flux (trying to slow down the increase).